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Physical Effects in Cavitating Flows

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A discussion of the physical effects in cavitation naturally divides itself into a part concerned with microscopic aspects and a part concerned with macroscopic aspects. The microscopic features of cavitating flows are related to the properties of nuclei in liquids and to the moderate tensile strengths which are usually encountered in flows. The macroscopic features are concerned with the growth of vapor or gaseous cavities from a small initial size and with their eventual collapse. The growth and collapse of vapor or gaseous cavities are macroscopic problems of a transient nature. There are macroscopic problems of a quasi-steady nature which are encountered in cavity flows—or in supercavitating flows, as they are sometimes called. We shall not consider these quasi-steady flows in any detail here.

TENSILE STRENGTH OF LIQUIDS

As is well known, the theoretical tensile strength of a liquid free of contaminants is very large—much larger than is ever observed. The theory of this tensile strength is statistical in nature so that one determines the probability of appearance of a small vapor bubble under a given tension (ref. 1). The formation of a bubble of radius R_0 requires an increase in free energy of $4\pi R_0^2\sigma$ where σ is the surface tension. As was recognized a long time ago by Gibbs, the pressure within the bubble exceeds the pressure in the liquid by $2\sigma/R_0$ so that the net increment in free energy is reduced to $4\pi R_0^2\sigma/3$. The probability, P , of appearance of a bubble is dominated by this free energy:

$$P \sim e^{-4\pi R_0^2\sigma/(3kT)} \quad (1)$$

In equation (1), k is the Boltzmann constant and T is the absolute temperature. It is apparent that this probability is very small even for large tensions. For example, with $\sigma = 72$ dynes/cm (water at 27°C), $\ln P = -73$ when $R_0 = 10^{-7}$ cm; the corresponding tension is 1400 atm.

While there is some reduction in the macroscopic value of the surface tension constant when the radius of curvature becomes very small (ref. 2), this reduction is not significant unless R_0 is less than 10^{-7} cm. We must conclude, therefore, that a homogeneous, pure liquid should have tensile strengths of the order of several thousand atmospheres. It is pertinent to remark that this large tensile strength would be predicted for liquids containing dissolved gases, so long, of course, as the gas is truly dissolved—that is to say, so long as the gas is distributed through the liquid on a molecular scale.

The conventional resolution of the disagreement between the theory of the tensile strength of a liquid and the experimental findings of much lower values is made with the introduction of "nuclei" in the liquid. Various properties of these nuclei have been proposed and we shall wish to discuss some of these here, but their function is to provide sites at which bubbles may grow from some effective size, R_0 , which is large enough to give a moderate value of $2\sigma/R_0$. In the statistical theory of the strength of a pure liquid, a cavity of radius R_0 must be formed where no nucleus exists through a statistical fluctuation. Such fluctuations are most improbable unless the spherical cavity is of the order of molecular dimensions. If the cavity in the liquid already exists at a larger radius, the corresponding tensile strength will be reduced.

One model which has been suggested for nuclei is that they are solid particles such as dust particles. It is easy to show (ref. 1) that such solid particles must not be wetted by the liquid if they are to serve as sites at which a macroscopic cavity is to grow. For water one would expect an abundance of hydrophobic dust particles, and, to give agreement with observations, "ordinary" water should have such particles with sizes at least as large as $R_0 \sim 10^{-4}$ cm; very carefully prepared "clean" water should have particles no larger than $R_0 \sim 10^{-5}$ cm or 10^{-6} cm.

Solid particles will tend to settle out from the liquid so that there should be some tendency for the tensile strength of ordinary water, for example, to rise with time if the sample is quiescent and protected from the atmosphere. Such an "aging" effect has been observed (ref. 3). It must be admitted, however, that this aging effect could be explained by undissolved air bubbles rising out of the liquid. The tensile strengths of very clean liquids should not show a significant aging effect since very small nuclei, say $R_0 \sim 10^{-6}$ cm, can be maintained in suspension by the mechanism of Brownian motion. In a macroscopic flow, much larger particles will be effectively maintained in the liquid since their settling velocities are so small.

So far we have not considered any effects of dissolved gases on the tensile strength of the liquid. In the application of a single tension pulse, or of a steady tension, this picture of the tensile strength as being determined by suspended, solid nuclei would indicate that there should be no

significant effect from dissolved gases. An increase in the effective size of solid nuclei by layers of gas adsorbed from solution would be unimportant since such a layer would have a thickness determined by the range of intermolecular forces. This range is of the order of 10^{-7} cm. There is, however, a macroscopic mechanism whereby dissolved gases can play a significant role; this mechanism is the surface tension resultant at the interface of three media: liquid, gas, and solid. The surface tension forces can be annulled, or even reversed, for such combinations so that undissolved pockets of gas can be stabilized in crevices of solid nuclei.

Undissolved bubbles of gas have been considered as possible nuclei from which macroscopic cavities can grow under reduced pressure. Such gas bubbles are, however, unstable. In an undersaturated solution the bubbles dissolve; even in a saturated solution gas bubbles will dissolve because of surface tension (ref. 4). In a sufficiently supersaturated solution, gas bubbles will grow indefinitely. While it must be acknowledged that the mass diffusion process which leads to collapse or growth of these gas bubbles is very slow compared with the dynamic process of vapor-bubble growth or collapse, it is difficult to explain a long-term modest tensile strength by means of undissolved gas bubbles unless they are attached to a solid surface. Such attachment of air bubbles with dimensions less than 10^{-4} cm has indeed been observed to exist on a rather permanent basis on hydrophobic particles in water (ref. 5).

There is a mechanism whereby the time required for the dissolution of very small free bubbles might be increased. Impurities, dissolved or distributed in the liquid, could tend to deposit on the surfaces of gas bubbles. This process would decrease the effective surface tension constant of the gas/liquid interface and would thereby represent a configuration of greater stability since the free energy would be decreased. The decrease in the surface tension would in itself lengthen the time for disappearance of the bubble and, in addition, could decrease the diffusion rate of gas through the bubble boundary into the liquid. An anomalous behavior in small oscillating gas bubbles has been observed (ref. 6) which might be due to such an adsorbed film. Some anomalous persistence of small bubbles has also been claimed (ref. 7) and might be similarly explained.

A liquid should show the same tensile strength for a single pulse of tension or for any nonrepetitive tension regardless of whether the nucleus is a solid, nonwetted particle or happens to be a small gas bubble. The growth from a nucleus to a macroscopic cavity under these conditions is so rapid that diffusion of dissolved permanent gas into the bubble from the solution does not take place.

If there are sufficiently long times available, diffusion of dissolved gases can play a central role in increasing the size of nuclei. Such long times are produced when a liquid is exposed to oscillating pressure fields. If the liquid has a concentration of dissolved gas, c_{∞} , which is the equilibrium

value for the liquid pressure P_0 , then a bubble of radius R_0 can gain mass when the liquid pressure undergoes periodic oscillations with angular frequency ω :

$$P(t) = P_0(1 + \epsilon \sin \omega t)$$

This process is known as rectified diffusion, and the rate of increase of mass in the bubble averaged over many oscillation cycles is (ref. 8)

$$\frac{dm}{dt} = \frac{8\pi}{3} D c_\infty R_0 \epsilon^2$$

where D is the coefficient of diffusivity of the gas in the liquid. Clearly, rectified diffusion can stabilize a gas bubble which would otherwise dissolve. A gas bubble of radius R_0 in a liquid at pressure P_0 , with equilibrium-dissolved concentration c_∞ , will lose mass at the following rate (ref. 4):

$$\frac{dm}{dt} \simeq -4\pi D \frac{2\sigma}{P_0} c_\infty$$

The threshold pressure amplitude for rectified diffusion which would just balance this loss is

$$\Delta P = P_{\max} - P_0 = P_0 \left(\frac{3}{2} \frac{2\sigma}{P_0 R_0} \right)^{1/2}$$

If ΔP exceeds this value, the bubble will grow—but at the slow rate characteristic of diffusion processes.

Another situation in which the long times required for the diffusion process are available is familiar in the flow of water over a submerged body so that the static pressure is reduced. The liquid may then be appreciably supersaturated, and a small bubble may grow on the surface. Such bubble growth has been observed in the boundary layer of a model in the working section of a water tunnel. When the bubble has become large enough, it leaves the surface of the body, enters the flow, and may serve as the nucleus for formation of a macroscopic vapor bubble.

It may be of interest to note that Einstein gave a theoretical connection between gas diffusivity in a liquid and the liquid viscosity:

$$D = \frac{kT}{6\pi\mu a}$$

where a is the radius of the diffusing molecule, T is the absolute temperature, and μ is the coefficient of viscosity. From this relation it is evident that a highly viscous liquid like glycerol should have a coefficient of

diffusion which is approximately 10^{-3} times that of water. The implications of such a small diffusion coefficient in a viscous liquid for gaseous cavitation are indeed interesting.

To summarize our view of the effective tensile strength of liquids, the small observed tensile strengths are due to the presence of nuclei which should for the most part consist of nonwetted solid particles. Dissolved gases should play only a secondary part in tensile strength and cavitation except where slow processes can be effective, and typical of these is rectified diffusion with oscillating pressures or the flow of supersaturated dissolved gas into a bubble attached to a solid.

DYNAMICS OF CAVITATION BUBBLES

Growth of Vapor Bubbles

In the growth of a vapor bubble, we may suppose that the rate of growth gives bubble boundary velocities which are small compared with the speed of sound in the vapor. It follows that the pressure within the bubble will be close to the equilibrium vapor pressure corresponding to the temperature in the liquid at the boundary. As the bubble grows, the latent heat of evaporation must be supplied at the liquid boundary. The associated cooling will reduce the vapor pressure and may therefore affect the rate of growth. We may easily find the conditions under which this cooling effect becomes important.

Let us suppose that a vapor bubble grows to a macroscopic size, R , in a time τ from some initial size $R_0 \ll R$. Then the total mass of vapor which is evaporated into the bubble is $(4\pi/3)R^3\rho'$ where ρ' is the vapor density. If L is the latent heat of evaporation per gram, then the total heat required is

$$Q = (4\pi/3)R^3\rho' L$$

This heat is taken out of a liquid layer around the bubble which has an effective thickness determined by the thermal diffusivity of the liquid, κ ,

$$\kappa = \frac{k}{\rho c}$$

where k is the thermal conductivity, ρ is the density, and c is the specific heat of the liquid. The thickness of the liquid layer is given approximately by

$$d \simeq (\kappa\tau)^{1/2}$$

We see that the estimate of the thickness is not sensitive to the choice of τ . The volume of the liquid layer is then of the order of magnitude

$4\pi R^2 d$, and the corresponding mass of liquid is $4\pi R^2 \rho d$. The temperature drop in this mass of liquid, ΔT , is then

$$\Delta T \sim \frac{(4\pi/3)R^3 \rho' L}{4\pi R^2 \rho c d} = \frac{R}{3d} \frac{\rho'}{\rho} \frac{L}{c}$$

The drop in vapor pressure, Δp_v , associated with this temperature decrease may be estimated from the Clausius-Clapeyron equation, which gives

$$\frac{dp_v}{dT} \sim \frac{LM}{BT^2} p_v$$

where M is the molecular weight of the vapor, B is the perfect gas constant, and T is the absolute temperature. This expression for the rate of change of vapor pressure with temperature comes from supposing that the vapor behaves like an ideal gas and, further, that $\rho \gg \rho'$. Both these approximations are fairly accurate. It follows that

$$\Delta p_v \sim \left(\frac{dp_v}{dT} \right) \Delta T$$

and finally

$$\frac{\Delta p_v}{p_v} \sim \frac{LM}{BT^2} \frac{R}{3d} \frac{\rho'}{\rho} \frac{L}{c} \quad (2)$$

The condition for a negligible effect from cooling by evaporation is that

$$\frac{\Delta p_v}{p_v} \ll 1$$

When this condition is satisfied, thermodynamic effects are unimportant in cavitation bubble growth. We then say that we have bubble growth in a "cold" liquid since it is evident that both ρ' and (dp_v/dT) decrease rapidly with temperature.

In a cold liquid, since thermodynamic effects are unimportant, we may disregard the energy equation and consider only the momentum equation. Since we have already supposed that the bubble growth rate is small compared with the speed of sound in the vapor, it will certainly be slow compared with the speed of sound in the liquid. The flow in the liquid is then essentially incompressible, and if, in addition, we take the growth to be spherically symmetric, the momentum equation integrates to the Bernoulli equation. From the Bernoulli equation one gets the equation of motion of the bubble boundary

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{p(R) - P_0}{\rho} \quad (3)$$

where P_0 is the pressure in the liquid at infinity and $p(R)$ is the pressure in the liquid at the bubble boundary. The boundary condition of continuity of stress across the bubble boundary gives

$$p(R) = p_v - \frac{2\sigma}{R} - 4\mu \frac{\dot{R}}{R} \quad (4)$$

The problem of spherical growth in a cold liquid is thus completely determined. We may observe that the left side of equation (3) may be written in the form

$$\frac{1}{2R^2\dot{R}} \frac{d}{dt} (R^3\dot{R}^2)$$

and one can easily get the asymptotic growth velocity for fixed P_0 :

$$\dot{R} \sim \left[\frac{2}{3} \frac{(p_v - P_0)}{\rho} \right]^{1/2} \quad (5)$$

since for large R the terms $2\sigma/R$ and $4\mu\dot{R}/R$ in $p(R)$ become unimportant.

When the condition $\Delta p_v \ll p_v$ is not met, the dynamics of vapor-bubble growth are strongly coupled to the energy equation, or the heat-flow equation. The physical situation now corresponds to tension in a "hot" liquid. We consider, for simplicity, the case in which the static pressure at infinity, P_0 , is constant, and we assume further that the temperature in the liquid is T_0 . The liquid must be superheated if a vapor bubble is to grow; that is, the vapor pressure at the liquid temperature, $p_v(T_0)$, exceeds the ambient pressure. We introduce the temperature T_b , which is the "boiling" temperature for the ambient pressure P_0 ; that is, T_b is the temperature such that $p_v(T_b) = P_0$. Then

$$p_v(T_0) > p_v(T_b) = P_0$$

We characterize the "hot" liquid case as that in which the cooling effect lowers the temperature in the bubble so that it approaches T_b as the bubble grows. When the cooling effect is so large, we have the limiting case in which inertial effects are unimportant so that the growth is controlled entirely by the rate of heat flow. Clearly, the cooling effect can never lower the temperature, T , within the bubble so far that $p_v(T) < P_0$. Growth cannot then proceed, and the heat inflow would cease together with any cooling. In the asymptotic case, we can expect that the temperature T within the bubble would exceed T_b by only a small amount, so that we may estimate the heat inflow requirement approximately by taking $T \simeq T_b$. The temperature in the liquid rises from its value at the bubble wall to T_0 in a distance of the order of magnitude $(\kappa t)^{1/2}$ at the time t . This distance is the heat diffusion length. The temperature

gradient in the liquid, then, is approximately $(T_0 - T_b)/(\kappa t)^{1/2}$, and the heat inflow into the bubble is

$$\dot{Q} \simeq 4\pi R^2 k \frac{(T_0 - T_b)}{(\kappa t)^{1/2}} \quad (6)$$

where $R = R(t)$. Now the heat requirement per unit time for evaporation is

$$\dot{Q} = L \frac{d}{dt} \left(\frac{4}{3} \pi R^3 \rho' \right) \simeq 4\pi R^2 \dot{R} L \rho' \quad (7)$$

If we equate these two values of \dot{Q} , we find the asymptotic growth speed

$$\dot{R} \simeq \frac{k}{L \rho'} \frac{(T_0 - T_b)}{(\kappa t)^{1/2}} \quad (8)$$

We see that the two limiting cases for growing vapor bubbles have very simple behavior described by equations (5) and (8). The intermediate situation of a "lukewarm" liquid does not have this simplicity, although we can be assured that the $R(t)$ trajectory must lie between those specified by the limiting curves just described. An accurate solution of the general case has analytical complications because the heat-flow equation is coupled to the momentum equation. The essential complication comes from the heat flow across a moving boundary $R(t)$, which is implicit in the problem (ref. 9).

Collapse of Vapor Bubbles

When the ambient pressure P_0 exceeds the vapor pressure p_* , so long as these pressures remain constant we may use the momentum equation (3) to determine the collapse velocity. If we suppose, for the moment, that p_* remains constant, and if, in addition, we neglect the effect of viscosity, then we easily find that

$$\dot{R}^2 = \frac{2(P_0 - p_*)}{3\rho} \left(\frac{R_0^3}{R^3} - 1 \right) + \frac{2\sigma}{\rho R} \left(\frac{R_0^2}{R^2} - 1 \right)$$

It follows that \dot{R} increases in magnitude like $R^{-3/2}$ as R approaches zero. Clearly, many of the simplifying assumptions which have been made cannot remain valid toward the end of the collapse motion.

Even in a cold liquid, a stage in the collapse will be reached in which the heat of condensation will produce a temperature rise so that p_* will increase. In this case, however, it may be shown by numerical treatment that the time for collapse and the trajectory for $R(t)$ are not greatly affected by this temperature rise. An additional effect which is of particular significance for cavitation or cold bubble collapse is the effect of compressibility, which has been studied by many authors.

There are evidently many complications which appear in the theory of the collapse in addition to the effect of compressibility. Among these are the instability in the spherical shape and the difficulty of determining the equation of state of the compressed vapor. All these difficulties appear only in the last stages of the collapse, when the radius is quite small. It is known, however, that a physical case of particular interest is that of the collapse of cavitation bubbles near a solid boundary. For this physical situation, the presence of a neighboring solid boundary changes the collapse behavior early in the bubble history before the difficulties characteristic of the late stage of collapse can appear. The bubble is deformed, and a jet is formed directed toward the solid boundary (ref. 10). This result not only simplifies the theoretical analysis by making it possible to avoid these difficulties, but also has important implications for the mechanism of cavitation damage.

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DISCUSSION

J. M. ROBERTSON (University of Illinois): In turbomachinery flows with liquids, cavitation may enter to cause noise, loss of machine effectiveness, vibration, or material erosion damage. The designer is faced with the questions of when and where cavitation will appear, how the flow is modified, and the nature of the cavity collapse. A clear and lucid picture has been given of the matter of tensile strength of liquids relevant to cavitation occurrence and to subsequent consideration of the transient dynamics of cavitation bubbles, both growth and collapse, as related to damage. And, through concentration on the basic physics, this presentation has been achieved without recourse to a single figure or complex mathematics.

In the matter of tensile strength, the author's review brings this matter down from the large values of pure liquids to the rather small values usually encountered in engineering. Concerning "dissolved gas" effects, I request the author's interpretation of the rather well-known experiments in which pressurization of nonpurified water led to much larger tensile strengths. My rough interpretation is that although the gas in the sample initially was not visually evident, it was only dissolved on pressurization.

The author suggests that often for cavitation to appear with oscillation, and in body flows, time must be available for gas diffusion into bubbles to occur. He notes that with highly viscous glycerol the diffusion would be significantly less than with water, thus implying a delay in cavitation occurrence. Cavitation experiments have dealt with glycerol, but concern was not with the initial appearance of cavitation. An even more viscous fluid (pluracol) was used by S. Sundaram (ref. D-1) for studies of cavitation inception (actually desinence) for a friction-type valve. The cavitation number was found to decrease by a factor of two with a doubling of the Prandtl number, which is directly proportional to viscosity.

In view of the use of fluids other than water in some machines, the discussion of bubble growth in "hot" liquids versus that in "cold" liquids and the resulting thermodynamic effect is germane to a host of turbomachinery flows. In an extensive review of cavitation damage, A. Thiruvengadam (ref. D-2) made a suggestive comparison of several fluids on this basis.

R. HICKLING (General Motors Research Laboratories): Professor Plesset has given a succinct summary of our present understanding of

some of the physical effects involved in the formation, growth, and collapse of bubbles. The theory that he presents allows a clear conceptualization of the processes involved.

The assumption of a uniform equilibrium behavior of the vapor inside the bubble permits considerable simplifications of the theory, but, in some cases, these may not be entirely realistic. It should be emphasized here that by a *nonequilibrium* behavior we mean deviations from the Clausius-Clapeyron equation—i.e., the assumption that the liquid and vapor phases are continuously in equilibrium with each other. Calculations for a bubble containing a permanent gas (ref. D-3) have shown that significant thermal gradients develop in the gas next to the cavity wall, even when the bubble wall is moving relatively slowly. A similar situation is expected to occur with a vapor. Such nonuniformity of temperature naturally has associated with it a nonequilibrium behavior in the vapor, and, under certain conditions, the interaction could become quite pronounced.

It has been argued that, because the thermal diffusivity of the vapor in the bubble is much greater than that of the surrounding liquid, the liquid presents a kind of thermal barrier to the flow of heat in and out of the bubble. It is reasoned that the vapor responds relatively quickly to thermal changes and consequently has a more uniform temperature distribution, while the liquid responds relatively slowly and has steep temperature gradients in the region close to the bubble wall. Surely this argument does not completely prove its case. It is indeed true that the thermal diffusion length of the vapor is an order of magnitude greater than the diffusion length in the liquid. However, the thermal capacity (density times specific heat) of the vapor is *two* orders of magnitude less than that of the liquid. Thus the temperature variations in the liquid phase should be an order of magnitude less than the temperature variations in the vapor. Therefore, one would expect the principal temperature gradients at the interface to lie within the vapor phase rather than the liquid. A nonuniform temperature in the vapor would appear to be the rule rather than the exception.

Having raised the specter of a nonuniform, nonequilibrium state inside the bubble, one next wonders what to do about it. Clearly this has to be considered if improvements are to be made in the theory. Recent attempts at this have involved rather gross assumptions and do not appear to be an advance on the basic work reported in Professor Plesset's paper.

PLESSET (author): Professor Robertson has asked me to comment on the "dissolved gas" effect which is found when a liquid is subjected to high pressure. After a liquid such as water has been so treated and when it is then tested under ordinary pressure, it shows a large tensile strength. This large tensile strength, however, gradually disappears as the liquid returns to its normal low tensile strength. I believe that this phenomenon is not primarily due to undissolved gas in the usual sense. I think that,

rather, it is due to the presence of solid particles which are not wetted by the liquid. These particles under ordinary conditions act as holes in the liquid and will therefore serve as "nuclei" or sites at which cavities can form with moderate tensions. When, on the other hand, the liquid is put under high pressure, the liquid will be forced into the surface of the solid particles, which will then no longer be unwetted and will no longer act as nuclei. Such particles, of course, may have originally stable layers of gas adsorbed on their surfaces, but such gas layers would be expected to be only a few molecules thick. Upon pressurization, this gas will also be forced into the surface of the solid with some liquid and may in this sense be dissolved. The gradual return to low tensile strength is to be interpreted as the diffusion of liquid out of the solid particles so that it again acts as a nucleus.

While Dr. Hickling has raised some subtle and interesting points in his discussion of my paper, I believe that his reservations regarding the present treatments of thermal problems in bubble dynamics are not justified. He suggests that nonequilibrium behavior in the liquid/vapor system will occur even when the interface moves slowly. This question has been considered in detail, and it has been shown that the vapor pressure at a liquid interface will have its equilibrium value (the "Clausius-Clapeyron" value) when the motion of the interface is small compared with the characteristic vaporization velocity c^* where

$$c^* = \frac{c}{(2\pi\gamma)^{1/2}}$$

Here c is the velocity of sound in the vapor and γ is the ratio of specific heats for the vapor. When the liquid interface is moving into the vapor with a velocity \dot{R} , the pressure in the vapor adjacent to the liquid, p' , is above the equilibrium value and is given by

$$p' = p_{eq} \frac{c^*}{c^* - \dot{R}}$$

where p_{eq} is the Clausius-Clapeyron value. When the interface is moving away from the vapor, one has

$$p' = p_{eq} \frac{c^*}{c^* + \dot{R}}$$

These expressions, which are accurate when \dot{R} is less than c^* , show that the deviations of p' from p_{eq} are unimportant for interface motions that are characterized by small Mach numbers. These slow motions are typical of those encountered in most cavitation and boiling situations.

We can also easily answer the question of the spatial variations in vapor pressure throughout a bubble of radius R . The vapor pressure at a bubble

wall, as we see from the expressions above, is the equilibrium vapor pressure corresponding to the liquid temperature at the wall, provided that \dot{R} is small compared to c^* . But this condition is a statement that the bubble wall has a motion with small Mach number. Such a motion of the vapor is essentially subsonic, incompressible motion. In this approximation it follows that the pressure in the vapor cannot have any appreciable spatial variation. The conclusion is that a bubble which has a moderate radial velocity, \dot{R} , has a spatially uniform vapor pressure which is given by the equilibrium vapor pressure corresponding to the temperature of the liquid interface.

We cannot yet conclude that the vapor density and vapor temperature are uniform throughout the bubble. Although the vapor pressure is uniform in the bubble, the vapor density and temperature in the interior of the bubble do not necessarily have the values that they assume at the boundary. It is clear, however, that both of these latter quantities must be spatially uniform if one of them is uniform, since the pressure, density, and temperature are connected by the equation of state. We can readily see that the vapor temperature is uniform for the moderate values of \dot{R} of present concern. This conclusion follows because the diffusion length will then be greater than R , and the temperature in the bubble, while it may vary with time, will not have appreciable spatial variation. Dr. Hickling is aware that the diffusion length will be large compared with R . It is of course well known that the coefficient of thermal diffusion is the only physical parameter which appears in the equation for heat flow, and, as Dr. Hickling points out, the thermal diffusivity of the vapor is much larger than that of the liquid, although the thermal conductivity of the vapor is appreciably less than that of the liquid. The only way that the thermal conductivity of the vapor or the liquid can enter into a problem, however, is through boundary conditions, and the boundary conditions should be considered with care since this is where Dr. Hickling is in error in his view of the problem. On the liquid side of the interface the heat flow per unit time is

$$-k_L 4\pi R^2 \left(\frac{\partial T}{\partial r} \right)_L$$

where k_L is the thermal conductivity of the liquid, $(\partial T / \partial r)_L$ is the temperature gradient in the liquid at the bubble wall, and $4\pi R^2$ is the area through which the heat flows. The boundary condition on the vapor side of the interface is *not*

$$-k_v 4\pi R^2 \left(\frac{\partial T}{\partial r} \right)_v,$$

as Dr. Hickling supposes. Rather, the boundary condition in the vapor is

$$L \frac{d}{dt} \left(\frac{4}{3} \pi R^3 \rho_v \right)$$

where L is the latent heat of evaporation or condensation. We see that the heat flow into an expanding bubble needed for latent heat of evaporation is supplied by a temperature gradient in the liquid, but we also see that no gradient is developed in the vapor. In the same way, for a shrinking bubble the latent heat of condensation of vapor is conducted away by a suitable gradient in the liquid without the development of a gradient in the vapor. Our conclusion is, finally, that a vapor bubble which expands or contracts at moderate rates will have a spatially uniform interior: uniform in vapor pressure, density, and temperature.

Dr. Hickling has also made reference to permanent gas bubbles in a liquid and again expresses concern that large temperature gradients are to be expected in the gas near the bubble boundary. As in the condensible vapor case, the thermal diffusivity of permanent gas is much larger than that of the liquid, so that the diffusion length may be much larger than that of the liquid. The physical situation is straightforward. When the diffusion length is large compared with the bubble radius—that is, for slow processes—the bubble temperature will remain isothermal. The fact that the thermal conductivity of the gas is small compared to that of the liquid is immaterial, since the temperature gradients in the liquid and in the gas are both essentially zero. The boundary condition at the bubble wall becomes trivial. The case of rapid motions of a permanent gas bubble is perhaps outside the scope of this discussion, but it may have some interest nevertheless. If the diffusion length in the gas becomes appreciably smaller than the bubble radius, the bubble motions will become adiabatic in some average sense. So far as the boundary condition in the liquid at the bubble wall is concerned, the temperature gradient is negligible; no appreciable heat flows into or out of the bubble. Gradients develop in the gas, and it is for this reason we must speak of an adiabatic behavior in an average sense.

I hope that these comments represent a clarification of a problem which has been beset by difficulties in understanding.

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